Pythagoras and Music

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The musician draws the bow across the violin string and immediately the inert atmosphere becomes vibrant and transparent, "…as if sound from non-spatial realms shines through a window into the world of space."[1] What is being demonstrated is a phenomenon known as the overtone series, in which any tone, played or sung, activates a column of mathematically-related notes which vibrate sympathetically with the sounded pitch and create resonance. Octaves throughout the universe respond, in a modern, scientific "music of the spheres," echoing the hypothesis that dates back to ancient times.

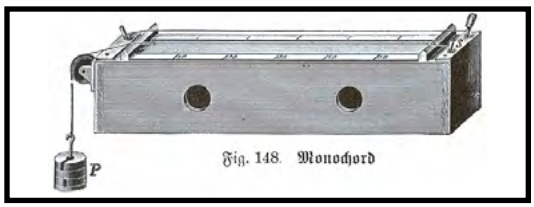
Until the Industrial Revolution, art, religion, and science were intertwined; the order of the universe was an ongoing study, later coming to fruition in the modern science of astronomy. But in the meantime, something fell away from science that had defined it for centuries—a connection with mysticism. Pythagoras—the Greek philosopher, initiate, and teacher—stood at the point of the marriage of music, science, and mysticism.[2] He was one of the first "scientists," and as an initiate, he asked deep questions of the universe.

*Gregor Reisch, A*rithmetica*, from* Margarita Philosophia *(1504). Pythagoras is on the right.*

Traveling to the centers of Babylonia and Egypt before settling in southern Italy, Pythagoras was likely exposed to ancient teachings about the power of number, as well as subjects which sensitized him to ask those important questions. He was known to be a person of great knowledge and psychic power. Due to his higher nature, legends grew up around him, such as an ability to travel to celestial realms and actually hear the music of the spheres. The school that he founded went on to take his oral teachings to even greater heights, influencing every future era of Western civilization. If we ask why Pythagoras's teachings and discoveries were so far-reaching, we arrive at the beginning point of the law that "everything vibrates"— a discovery which became a turning-point for a new understanding of the universe.

The Discovery of Musical Law

Pythagoras's mind, alive to possibilities, came upon a very simple theorem that had cosmic value. The legend is that Pythagoras, while walking past a blacksmith's shop, heard different pitches being emitted from the striking of the anvils. What is said to have gone through his mind was that the variation in pitches was possibly created by the different weights of the hammers. This story, possibly symbolically inspired by the legend of a magical blacksmith's hammer, may have a basis in fact;[3] at any rate, Pythagoras began to experiment with musical overtones and ratios, which led to one of the most important discoveries of all time.



Monochord from Bibliothek allgemeinen und praktischen Wissens für Militäranwärter, *Volume 3, 1905.*

In his search to determine interval ratios in music (an interval being both the space and the relationship between two sounding notes), Pythagoras employed the lyre and the monochord, a one-stringed instrument he may have invented, which featured frets on the fingerboard at various lengths. By stopping the string exactly at the halfway point, he produced an octave, or a ratio of 1:2. By dividing the string into various other lengths, intervals of the fourth and fifth were produced, and so on.[4] Pythagoras and his followers conceived of the universe as a vast lyre, in which each planet, vibrating at a specific pitch, in relationships similar to the stopping of the monochord's string, harmonized with other heavenly bodies to create a "music of the spheres," a concept which remained viable for centuries. Even though his theory was primitive, it serves to give us a picture which was later developed by philosophers such as Boethius, Johannes Kepler, the Rosicrucian Robert Fludd, and, in contemporary times, by scientists working with quantum relationships.

The theories set forth by Pythagoras are complex to those uninitiated into mathematical and musical analysis, but certain concepts are important to set forth here. Nicomachus of Gerasa, a theorist in the first or second century CE, was an authority on Pythagoras and called himself a Pythagorean. In his *Manual of Harmonics*, he models his explanations of intervals, numbers, and the music of the spheres on Pythagoras's teachings as passed down through the years, and it is a good source from which to explain some basic concepts.[5]

The Pythagoreans found that the speed of vibration and the size of the sound-producing body were the factors in music that were regulated by number. A modern example would be the stringed bass, tuned to the lowest notes due to its size. Sound was said to be produced by percussion (striking), followed by a vibration in the air, which was then received by the ear and carried, in Plato's words, "to the brain and the blood and transmitted to the soul."[6] The theory was that the vibrational frequency of a stretched string is inversely proportional to its length.[7] This basic statement, despite the fact that the Pythagoreans had no way of actually measuring the vibrations of tones—so their method of assigning numerical values could not be relied on—laid the groundwork for the development of the science of acoustical physics. By assigning mathematical data as a basis for harmonious sound, Pythagoras was going

Franchinus Gaffurius, Theorica musicae *(1492): Pythagoras exploring harmony and ratio with various musical instruments.*

A detail from The School of Athens *by Raphael (1511), showing Pythagoras at work, surrounded by Archimedes at right, Averroes at top left, and Anaximander at middle left.*

against the persuasion of the day that pleasing harmony was merely a matter of taste and instinct.

The Perfect Octave Creates *Harmonia*

Working with his seven-stringed lyre, and thinking of the divisions of the strings that he had discovered, Pythagoras realized that for the relationships to be complete and balanced, the perfect interval of an octave (e.g., C^1-C^2) must be part of the existing scale. Because he had no way of numerically measuring the exactness of the divisions, we have to assume that he heard the sameness of the octave and "saw" the nodes that characterized the overtones.[8] He then added an eighth string to the lyre to create this octave, an action not easily condoned at the time, as Greek society held the number seven as sacred, and the addition of the octave disturbed the symbolism of the modes and the seven planets. However, Pythagoras's standing in the community and in the minds of his followers neutralized any censure that might have ensued.[9]

The original scale of seven notes contained a half step (the smallest interval in Western music) after the fourth tone.

When the octave was added, the interval that was created between the fifth note and the octave would be dissonant and offensive to Greek ears. Although this theory is controversial, it is assumed that what Pythagoras did technically was to change the fifth note by making it a half step higher. This caused the two four-note halves of the scale (known as tetrachords) to be separated by a whole step instead of a half step. This action preserved the value of the tetrachords, yet created the octave he was looking for.

The original seven-note scale looked like this:[10]

E F G A B♭ C D

half step

If the octave E were simply added as the eighth note, the interval between the B♭ and upper E would be a tritone (three full steps), which is dissonant:

E F G A B♭ C D E

dissonance

So Pythagoras merely changed the B♭ to a B♮, thereby creating a whole step between the tetrachords.

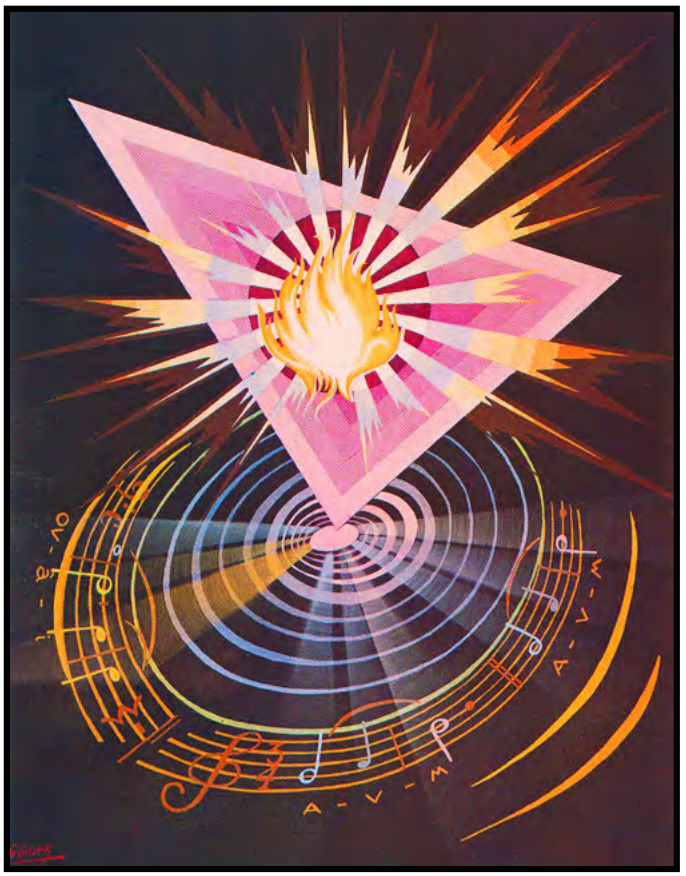
E F G A B♮ C D E

whole step

Now the B♮-E relationship was a consonant perfect fourth. This brilliant move changed history. Philosophically and musically, what he did was to create what he termed *harmonia* (unity, "fitting together") within the scale. The understanding of harmonia would teach humankind immutable laws and appeal to the higher, rational intellect; it balanced the limitless and the limited, heaven and Earth.[11] Adding the octave implied more than just a re-sounding of the primary tone at a higher level; the octave is a completion of the scale and of all the intervals therein. The twentieth-century philosopher Rudolf Steiner, in teaching about the intervals, stated that ". . . the feeling for the octave brings us to find our own self on a higher level." In his perception, the

true experience of the octave will "become a new form of proving the existence of God [the Divine]."[12]

So Pythagoras's experiment altered music for all time, made possible the Greek scale systems, which later evolved into the medieval modal scales and our modern major and minor scales, established the basis for all future music theory work, and gave us an interval for the future. Flora Levin's commentary on Nicomachus's manual surmises: "As a musical act, [adding the octave] was sublimely simple; as a mathematical production, however, it was incalculably profound."[13]



Nicomedes Gómez, The Creative Word.

Through the discovery of musical laws, and that there is an order behind musical sounds, Pythagoras and his followers connected to the thought that the same order and relationship is found in nature and the universe:[14] "Music was number, and the cosmos was music."[15] Pythagoras as scientist and initiate experienced this revelation in the deepest sense of the word: suddenly a doorway was opened into a mystical understanding of the universe. The Pythagoreans realized that this mathematical order, or hidden pattern which was the basis of musical sounds, lay behind everything in nature and the cosmos. This was the first time that such a conscious connection had been established.[16]

Pythagoras's discovery of musical ratios and overtones provided him with the basis for his mathematical work; his vision of intervals as mathematical ideals symbolizing universal harmony or divine substance moved the cosmos from a poetic, visual phenomenon to a scientific conception that spawned centuries of further development in mathematics, music, astronomy, and cosmogony.

The Therapeutic Value of Music

A legend reported by Boethius (480-524 CE) states that Pythagoras, upon hearing of a youth who had been jilted by his lover and was preparing to set her house on fire, determined that the youth was under the influence of a certain musical mode (scale). By suggesting that he change his tune and employ a melody based on an alternate scale, Pythagoras was able to restore the youth to a state of calmness.[17] Whether this story is true or not, Pythagoras was one of the first musicians to recognize the therapeutic power of music. His work with the mathematical properties and ratios of musical intervals convinced him that the music of the human organism

1:1 C	2:1 C	3:1 G	4:1 C	5:1 E	6:1 G	7:1 Bb	8:1 C	9:1 D	10:1 E	11:1 Gb	12:1 G	13:1 Ab	14:1 Bb	15:1 B	16:1 C
1:2 C	2:2 C	3:2 G	4:2 C	5:2 E	6:2 G	7:2 Bb	8:2 C	9:2 D	10:2 E	11:2 Gb	12:2 G	13:2 Ab	14:2 Bb	15:2 B	16:2 C
1:3 F	2:3 F	3:3 C	4:3 F	5:3 A	6:3 C	7:3 Eb	8:3 F	9:3 G	10:3 A	11:3 B	12:3 C	13:3 Db	14:3 Eb	15:3 E	16:3 F
1:4 C	2:4 C	3:4 G	4:4 C	5:4 E	6:4 G	7:4 Bb	8:4 C	9:4 D	10:4 E	11:4 Gb	12:4 G	13:4 Ab	14:4 Bb	15:4 B	16:4 C
1:5 Ab	2:5 Ab	3:5 Eb	4:5 Ab	5:5 C	6:5 Eb	7:5 Gb	8:5 Ab	9:5 Bb	10:5 C	11:5 D	12:5 Eb	13:5 F	14:5 Gb	15:5 G	16:5 Ab
1:6 F	2:6 F	3:6 C	4:6 F	5:6 A	6:6 C	7:6 Eb	8:6 F	9:6 G	10:6 A	11:6 B	12:6 C	13:6 Db	14:6 Eb	15:6 E	16:6 F
1:7 D	2:7 D	3:7 A	4:7 D	5:7 Gb	6:7 A	7:7 C	8:7 D	9:7 E	10:7 Gb	11:7 Ab	12:7 A	13:7 B	14:7 C	15:7 Db	16:7 D
1:8 C	2:8 C	3:8 G	4:8 C	5:8 E	6:8 G	7:8 Bb	8:8 C	9:8 D	10:8 E	11:8 Gb	12:8 G	13:8 Ab	14:8 Bb	15:8 B	16:8 C
1:9 Bb	2:9 Bb	3:9 F	4:9 Bb	5:9 D	6:9 F	7:9 Ab	8:9 Bb	9:9 C	10:9 D	11:9 Eb	12:9 F	13:9 Gb	14:9 Ab	15:9 A	16:9 Bb
1:10 Ab	2:10 Ab	3:10 Eb	4:10 Ab	5:10 C	6:10 Eb	7:10 Gb	8:10 Ab	9:10 Bb	10:10 C	11:10 D	12:10 Eb	13:10 F	14:10 Gb	15:10 G	16:10 Ab
1:11 Gb	2:11 Gb	3:11 Db	4:11 Gb	5:11 Bb	6:11 Db	7:11 E	8:11 Gb	9:11 A	10:11 Bb	11:11 C	12:11 Db	13:11 Eb	14:11 E	15:11 F	16:11 Gb
1:12 F	2:12 F	3:12 C	4:12 F	5:12 A	6:12 C	7:12 Eb	8:12 F	9:12 G	10:12 A	11:12 B	12:12 C	13:12 Db	14:12 Eb	15:12 E	16:12 F
1:13 E	2:13 E	3:13 B	4:13 E	5:13 G	6:13 B	7:13 Db	8:13 E	9:13 Gb	10:13 G	11:13 A	12:13 B	13:13 C	14:13 Db	15:13 D	16:13 E
1:14 D	2:14 D	3:14 A	4:14 D	5:14 Gb	6:14 A	7:14 C	8:14 D	9:14 E	10:14 Gb	11:14 Ab	12:14 A	13:14 B	14:14 C	15:14 Db	16:14 D
1:15 Db	2:15 Db	3:15 Ab	4:15 Db	5:15 F	6:15 Ab	7:15 B	8:15 Db	9:15 Eb	10:15 F	11:15 G	12:15 Ab	13:15 Bb	14:15 B	15:15 C	16:15 Db
1:16 C	2:16 C	3:16 G	4:16 C	5:16 E	6:16 G	7:16 Bb	8:16 C	9:16 D	10:16 E	11:16 Gb	12:16 G	13:16 Ab	14:16 Bb	15:16 B	16:16 C

Pythagorean Table (Lambdoma).

would respond in various ways to these relationships; for the Pythagoreans, "both music and the soul share a basis in number."[18] The Greek modes used at that time were distinguished primarily by their intervallic structures; each was thought to have a positive or negative effect on the human psyche. Consequently, certain rhythms, scales, and songs were used to heal the body and soothe the passions.

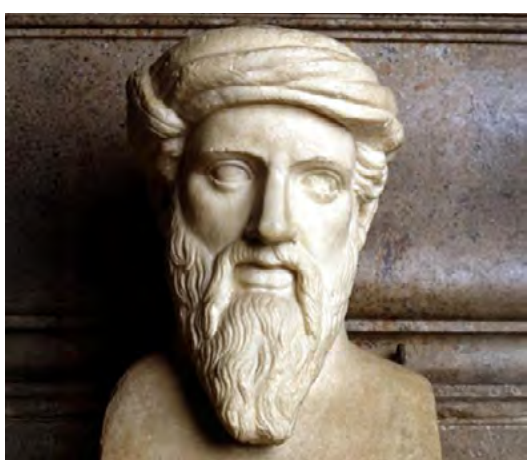
The Pythagoreans were said to have musical exercises for sleeping and waking, and to stimulate certain moods for work and relaxation. Later, Plato (ca. 427-347 BCE), who was strongly influenced by the Pythagorean doctrine of universal number, would take up this concept and make it into a science, prescribing musical modes for warriors, women, and various illnesses and passions.

Plato introduced Pythagoras's vision of a musical cosmos into the mainstream, which would result in it becoming a standard in the Greek worldview, and eventually in that of the entire civilization of Western thought. Pythagorean principles are the basis of much of musical and mathematical study, and the strength and depth of his discoveries changed the way the world perceived itself, even up to our times. However, his influence goes beyond the world of science and mathematics; it penetrates into the mystical side of number and music. A principal teaching of the Pythagorean School was that the Divine is universal harmony, perceived through number.[19]

Albert von Thimus in the nineteenth century used Pythagoras's concepts to create a "Pythagorean Table," which

mathematically "explains" the effect of music on the universe and on the human being.[20] Working from an ancient treatise, von Thimus and his colleagues believed that they had come upon the "fundamental diagram of the lost science of Harmonics, hinted at by Plato… as the culmination of all learning, but never revealed publicly."[21] The table projects the universe; although mathematically complex, it can be simplified by stating that each rational fraction and integer is arrived at as an intersection of an overtone and an undertone row. A tone is projected as a created "being," each being manifesting number and note. All beings have the same root; the original tone is 1/1 = the Divine. If we take the table's calculations beyond its boundaries, we arrive at 0/0, the point which sounds no tone: the Unmanifest, the Absolute, Mind, and Silence. Pythagoras's theorems thus projected point toward the future of science as it continues to ask questions of the universe.

Pythagoras's experience as an initiate, scientist, musician, and mystic made him singularly qualified to explore the mysteries of the universe through music and number. As Guy Murchie, writing on the music of the spheres, concludes, "Nature has a beautiful simplicity of order. And the intuitions of Pythagoras . . . are proving substantially justified."[22]



A bust of Pythagoras made from a Greek original of the middle of the fifth century BCE.

Endnotes

[1] Frits Julius, *Sound Between Matter and Spirit* (Chestnut Ridge, NY: Mercury Press, 1993), 24.

[2] Jamie James, *The Music of the Spheres: Music, Science, and the Natural Order of the Universe* (New York: Copernicus, an imprint of Springer-Verlag New York, by arrangement with Grove Press, 1993), 4.

[3] Kitty Ferguson, *The Music of Pythagoras: How An Ancient Brotherhood Cracked the Code of the Universe and Lit the Path from Antiquity to Outer Space* (New York: Walker, 2008), 67.

[4] For a fine explanation of musical intervals, which are key to the understanding of Pythagoras's discovery, see James, *Music of the Spheres*, 33-35.

[5] See *The Manual of Harmonics of Nicomachus the Pythagorean,* translation and commentary by Flora R. Levin, (Grand Rapids, MI: Phanes Press, 1994), passim.

[6] From the *Timaeus*, quoted in Nicomachus, *Manual of Harmonics*, 65.

[7] Nicomachus, Manual of Harmonics, 66.

[8] Guy Murchie, *Music of the Spheres: The Material Universe–From Atom to Quasar, Simply Explained* (New York: Dover, 1967), II: 383-384.

[9] Nicomachus, *Manual of Harmonics*, 77.

[10] Ibid, Basic chart, my annotations.

[11] Ferguson, *Music of Pythagoras*, 107; an Aristotelian concept.

[12] Rudolf Steiner, *The Inner Nature of Music and the Experience of Tone* (Spring Valley, NY: Anthroposophic Press, 1983), 48, 56-57.

[13] Nicomachus, *Manual of Harmonics*, 77.

[14] Ferguson, *Music of Pythagoras*, 65.

[15] James, *Music of the Spheres*, 31.

[16] Ferguson, *Music of Pythagoras*, 65.

[17] Boethius, "From the *De institutione musica*," in *Source Readings in Music History: Antiquity and the Middle Ages*, ed. Oliver Strunk, 79-86; further referred to by Joscelyn Godwin in *Harmonies of Heaven and Earth* (Rochester, VT: Inner Traditions, 1987), 30.

[18] Godwin, *Harmonies of Heaven*, 29.

[19] Edouard Schuré, *The Great Initiates* (West Nyack, NY: St. George Books, 1961), 307.

[20] Godwin, *Harmonies of Heaven*, 190-192. Descriptions of the Pythagorean table were inspired by Dr. Godwin's explanation.

[21] Ibid, 190.

[22] Murchie, *Music of the Spheres*, II: 598.